Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Mathematics-I

Subject Code : 4SC01MTC1 **Branch: B. Sc.(All)**

Semester : 1 **Date :25/04/2016** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Find the eigen values of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$. (02)

b) Evaluate $\lim_{x\to 0} \frac{1-\cos x}{x^2}$. (02)

c) Find the center and radius of the sphere $x^2 + y^2 + z^2 - 4x - 2y - 6z - 11 = 0$. (02)

d) Check whether the following differential equation is exact or not: (02)
$(1 + 2xy\cos x^2 - 2xy)dx + (\sin x^2 - x^2)dy = 0$.

e) Express the equation $x^2 + y^2 = 1$ into the polar form. (02)

f) Write down the series expansion of e^x in powers of x. (02)

g) State Roll's mean value theorem. (02)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) State Cayley-Hamilton theorem and verify it for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Also find A^{-1}. **(07)**

b) State and prove Leibnitz's theorem. **(07)**

Q-3 **Attempt all questions** **(14)**

a) Test for consistency and solve **(07)**
$5x + 3y + 7z = 4;\ \ 3x + 26y + 2z = 9;\ \ 7x + 2y + 10z = 5.$

b) State and prove Cauchy mean value theorem. **(07)**

Q-4 **Attempt all questions** **(14)**

a) State standard form of a linear equation of the first order and solve **(07)**
$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x + 1)$.

b) Apply Gauss elimination method to solve the equations **(07)**

$x + 4y - z = -5;\ \ x + y - 6z = -12;\ \ 3x - y - z = 4.$

Q-5 **Attempt all questions** **(14)**

a) Expand $\log x$ in powers of $(x - 1)$ and hence evaluate $\log 1.1$ correct to four decimal places. **(07)**

b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(07)**

Q-6 **Attempt all questions** **(14)**

a) If $y = e^{a \sin^{-1} x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. Hence find the value of y_n when $x = 0$. **(07)**

b) Solve the following system of equations using Cramer's rule: **(07)**
$3x + y + 2z = 3;\ \ 2x - 3y - z = -3;\ \ x + 2y + z = 4.$

Q-7 **Attempt all questions** **(14)**

a) Define: rank of the matrix and find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. **(07)**

b) Define: Hermitian matrix. If $A = \begin{bmatrix} 2 + i & 3 & -1 + 3i \\ -5 & i & 4 - 2i \end{bmatrix}$, show that AA^* is a Hermitian matrix, where A^* is conjugate transpose of A. **(07)**

Q-8 **Attempt all questions** **(14)**

a) Solve: $(x^2 - y^2)dx - xy\,dy = 0$. **(07)**

b) Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form. **(07)**